REVISION OF THE TRADITIONAL INDIAN PLANETARY MODEL BY NĪLAKAṆṬHA SOMASUTVAN (c. 1500 AD)[1]

It is now generally recognized that the Kerala School of Indian astronomy[2], starting with Mādhava of Saṅgamagrāma in the fourteenth century, made important contributions to mathematical analysis much before this subject developed in Europe. The Kerala astronomers derived infinite series for π, sine and cosine functions and also developed fast convergent approximations to them.[3]

Here, we shall show that the Kerala School also made equally significant discoveries in astronomy, in particular, planetary theory. Mādhava's disciple Parameśvara of Vatasseri (c.1380–1460) is reputed to have made continuous and careful observations for a period of over fifty-five years. He is famous as the originator of the *Dṛig-gaṇita* system, which replaced the older *Parahita* system. Nīlakaṇṭha Somasutvan of Trikkantiyur (c.1444-1550), the disciple of Parameśvara's son Dāmodara, carried out an even more fundamental revision of the traditional planetary theory. In his treatise *Tantrasaṅgraha* (c.1500), Nīlakaṇṭha presents a major revision of the earlier Indian planetary model for the interior planets Mercury and Venus. This led Nīlakaṇṭha to a much better formulation of the equation of centre and the latitude of these planets than was available either in the earlier Indian works or in the Islamic or the Greco-European traditions of astronomy till the work of Kepler, which was to come more than a hundred years later.

Nīlakaṇṭha was the first savant in the history of astronomy to clearly deduce from his computational scheme (and not from any speculative or cosmological argument) that the interior planets go around the Sun and the period of their motion around Sun is also the period of their latitudinal motion. He explains in his *Āryabhaṭīyabhāṣya* that the Earth is

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[1]Much of the material of this essay is based on the following sources, which may be consulted for further details: (i) K. Ramasubramanian, M. D. Srinivas and M. S. Sriram, 'Modification of the Earlier Indian Planetary Theory by the Kerala Astronomers (c.1500 AD) and the Implied Heliocentric Picture of Planetary Motion', Current Science 66 , 784-790, 1994; (ii) M. S. Sriram, K. Ramasubramanian and M D Srinivas (eds.), *500 Years of Tantrasaṅgraha*: *A Landmark in the History of Astronomy*, Shimla 2002, p.29-102.

[2]For the Kerala School of Astronomy, see for instance, K.V.Sarma, *A Bibliography of Kerala and Kerala-based Astronomy and Astrology*, Hoshiarpur 1972; K.V.Sarma, *A History of the Kerala School of Hindu Astronomy*, Hoshiarpur 1972.

[3]See for example: C.M. Whish, Trans. R. Asiatic Soc. 3, 509, 1835; K. Mukunda Marar, Teacher's Magazine 15, 28-34, 1940; K. Mukunda Marar and C. T. Rajagopal, J.B.B.R.A.S. 20, 65-82, 1944; C. T. Rajagopal, Scr. Math. 15, 201-209, 1949; C. T. Rajagopal and A. Venkataraman, J.R.A.S.B. 15, 1-13, 1949; C. T. Rajagopal and T. V. V. Aiyar, Scr. Math. 17, 65-74, 1951; C.T.Rajagopal and T.V.V.Aiyar, Scr. Math. 18, 25-30, 1952; C.T.Rajagopal and M.S.Rangachari, Arch. for Hist. of Ex. Sc. 18, 89-101, 1978; C. T. Rajagopal and M. S. Rangachari, Arch. for Hist. of Ex. Sc. 35(2), 91-99, 1986; T. Hayashi, T.Kusuba and M.Yano, Centauras, 33, 149-174, 1990; Ranjan Roy, Math. Mag. 63, 291-306, 1990; V.J.Katz, Mag. 68, 163-174, 1995; C.K.Raju, Phil. East and West 51, 325-362, 2001; D.F.Almeida, J.K.John and A.Zadorozhnyy, J. Nat. Geo. 20, 77-104, 2001; D. Bressoud, College Math. J. 33, 2-13, 2002. For an overview of the Kerala tradition of mathematics, see, S. Parameswaran, *The Golden Age of Indian Mathematics*, Kochi 1998; G.C.Joseph, *The Crest of the Peacock*: *Non-European Roots of Mathematics,* 2nd Ed., Princeton 2000.

not circumscribed by the orbit of the interior planets, Mercury and Venus; and the mean period of motion in longitude of these planets around the Earth is the same as that of the Sun, precisely because they are being carried around the Earth by the Sun. In his works, *Golasāra* and *Siddhāntadarpaṇa*, Nīlakaṇṭha describes the geometrical picture of planetary motion that follows from his revised model, where the five planets Mercury, Venus, Mars, Jupiter and Saturn move in eccentric orbits around the mean Sun, which in turn goes around the Earth. Most of the Kerala astronomers who succeeded Nīlakaṇṭha, such as Jyesṣṭhadeva, Acyuta Piṣāraṭi, Putumana Somayāji, etc. seem to have adopted this planetary model.

I. THE CONVENTIONAL PLANETARY MODEL OF INDIAN ASTRONOMY

In the Indian astronomical tradition, at least from the time of Āryabhaṭa (499 AD), the procedure for calculating the geocentric longitudes of the five planets, Mercury, Venus, Mars, Jupiter and Saturn involves essentially the following steps.[4] First, the mean longitude (called the *madhyama-graha*) is calculated for the desired day by computing the number of mean civil days elapsed since the epoch (this number is called the *ahargaṇa*) and multiplying it by the mean daily motion of the planet. Then two corrections namely the *manda-saṁskāra* and *śīghra- saṁskāra* are applied to the mean planet to obtain the true longitude.

In the case of the exterior planets, Mars, Jupiter and Saturn, the *manda- saṁskāra* is equivalent to taking into account the eccentricity of the planet's orbit around the Sun. Different computational schemes for the *manda- saṁskāra* are discussed in Indian astronomical literature. However, the *manda* correction in all these schemes coincides, to first order in eccentricity, with the equation of centre currently calculated in astronomy. The *manda*-corrected mean longitude is called *mandasphuṭa-graha*. For the exterior planets, the *mandasphuṭa-graha* is the same as the true heliocentric longitude.

The *śīghra- saṁskāra* is applied to this *mandasphuṭa-graha* to obtain the true geocentric longitude known as *sphuṭa-graha*. The *śīghra* correction is equivalent to converting the heliocentric longitude into the geocentric longitude. The exterior and interior planets are treated differently in applying this correction, and we take them up one after the other.

[4]For a general review of Indian astronomy, see D.A. Somayaji, *A Critical Study of Ancient Hindu Astronomy*, Dharwar 1972; S.N. Sen and K.S. Shukla (eds.), *A History of Indian Astronomy*, New Delhi 1985; B.V. Subbarayappa, and K.V. Sarma (eds.), *Indian Astronomy*: *A Source Book*, Bombay 1985; S.Balachandra Rao, *Indian Astronomy*: *An Introduction,* Hyderabad 2000.

Exterior planets

For the exterior planets, Mars, Jupiter and Saturn, the mean heliocentric sidereal period is identical with the mean geocentric sidereal period. Thus, the mean longitude calculated prior to the *manda- saṁskāra* is the same as the mean heliocentric longitude of the planet as we understand today. As the *manda- saṁskāra*, or the equation of centre, is applied to this longitude to obtain the *mandasphuṭa-graha*, the latter will be true heliocentric longitude of the planet.

The *śīghra- saṁskāra* for the exterior planets can be explained with reference to Figure 1. Longitudes are always measured in Indian astronomy with respect to a fixed point in the Zodiac known as the *Nirayana Meṣādi* denoted by A in the figure. E is the Earth and P the planet. The mean Sun S is referred to as the *śīghrocca* for exterior planets. We have

$\angle$ASP =	θ_{mS}	=	*Mandasphuṭa*
$\angle$AES =	θ_S	=	Longitude of *śīghrocca* (mean Sun)
$\angle$AEP =	θ	=	True geocentric longitude of the Planet

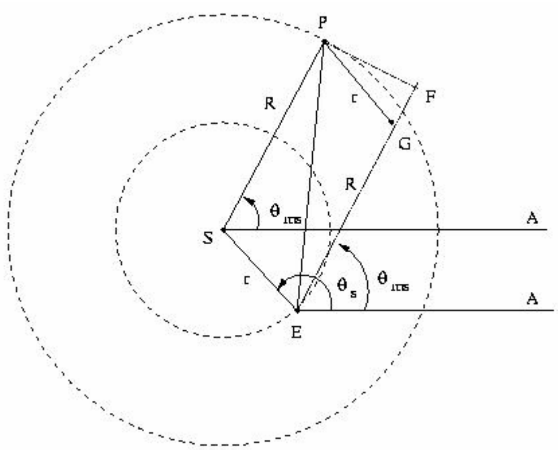

Figure 1: *Śīghra* correction for Exterior Planets

The difference between the longitudes of the *śīghrocca* and the *mandasphuṭa*, namely,

$$\sigma = \theta_S - \theta_{mS} \tag{1}$$

is called the *śīghra-kendra* (anomaly of conjunction) in Indian astronomy. From the triangle EPS we can easily obtain the result

$$\sin(\theta - \theta_{mS}) = \frac{r \sin\sigma}{[(R + r\cos\sigma)^2 + r^2 \sin^2\sigma]^{1/2}} \tag{2}$$

which is the *śīghra* correction formula given by Indian astronomers to calculate the geocentric longitude of an exterior planet.

From the figure it is clear that the *śīghra-saṁskāra* transforms the true heliocentric longitudes into true geocentric longitudes. This will work only if *r/R* is equal to the ratio of the Earth-Sun and Planet-Sun distances and is indeed very nearly so in the Indian texts. But equation (2) is still an approximation as it is based upon the identification of the mean Sun with the true Sun.

Interior planets

For the interior planets Mercury and Venus, ancient Indian astronomers, at least from the time of Āryabhaṭa, took the mean Sun as the *madhyama-graha* or the mean planet. For these planets, the mean heliocentric sidereal period is the period of revolution of the planet around the Sun, while the mean geocentric sidereal period is the same as that of the Sun. The ancient astronomers prescribed the application of *manda* correction or the equation of centre characteristic of the planet, to the mean Sun, instead of the mean heliocentric planet as is done in the currently accepted model of the Solar System. However, the ancient Indian astronomers also introduced the notion of the *śīghrocca* for these planets whose period is the same as the mean heliocentric sidereal period of these planets. Thus, in the case of the interior planets, it is the longitude of the *śīghrocca* which will be the same as the mean heliocentric longitude of the planet as understood in the currently accepted model for the Solar System.

The *śīghra- saṁskāra* for the interior planets can be explained with reference to Figure 2. Here E is the Earth and S (*manda*-corrected mean Sun) is the *mandasphuṭa-graha* and P corresponds to the planet. We have,

$\angle$AES =	θ_{mS}	=	*Mandasphuṭa*
$\angle$ASP =	θ_S	=	Longitude of *śīghrocca*
$\angle$AEP =	θ	=	True geocentric longitude of the Planet

Again, the *śīghra-kendra* is defined as the difference between the *śīghrocca* and the *mandasphuṭa-graha* as in (1). Thus, from the triangle EPS we get the same formula

$$\sin(\theta - \theta_{mS}) = \frac{r \sin\sigma}{[(R + r\cos\sigma)^2 + r^2 \sin^2\sigma]^{1/2}} \tag{3}$$

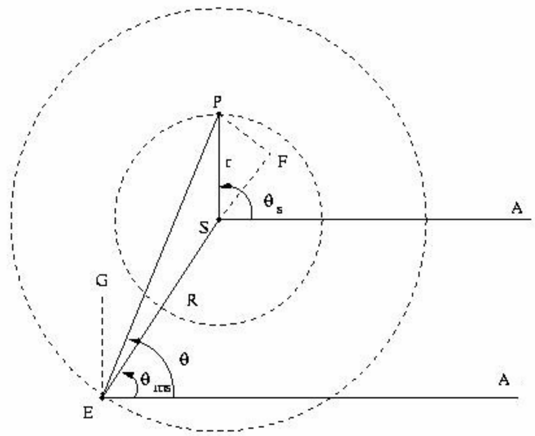

Figure 2: *Śīghra* correction for Interior Planets

which is the *śīghra* correction given in the earlier Indian texts to calculate the geocentric longitude of an interior planet. For the interior planets also, the value specified for r/R is very nearly equal to the ratio of the Planet-Sun and Earth-Sun distances. In Table 1, we give Āryabhaṭa's values for both the exterior and interior planets along with the modern values based on the mean Earth-Sun and Sun-Planet distances.

Table 1: Comparison of *r/R* in *Āryabhaṭīya* with modern values

Planet	*Āryabhaṭīya*	Modern value[5]
Mercury	0.361 to 0.387	0.387
Venus	0.712 to 0.737	0.723
Mars	0.637 to 0.662	0.656
Jupiter	0.187 to 0.200	0.192
Saturn	0.114 to 0.162	0.105.

Since the *manda* correction or equation of centre for an interior planet was applied to the longitude of the mean Sun instead of the mean heliocentric longitude of the planet, the accuracy of the computed longitudes of the interior planets according to the ancient Indian planetary models would not have been as good as that achieved for the exterior planets.

II. COMPUTATION OF THE PLANETARY LATITUDES

Planetary latitudes (called *vikṣepa* in Indian astronomy) play an important role in the prediction of planetary conjunctions, occultation of stars by planets, etc. In Figure 3, P denotes the planet moving in an orbit inclined at angle *i* to the ecliptic, intersecting the ecliptic at the point N, the node (called *pāta* in Indian astronomy). If β is the latitude of

[5]Ratio of the mean values of Earth-Sun and Planet-Sun distances for the exterior planets and the inverse ratio for the interior planets

the planet, θ_H its heliocentric longitude, and θ_o the heliocentric longitude of the node, then for small *i* we have

$$\sin \beta = \sin i \ \sin(\theta_H - \theta_o) \simeq i \sin(\theta_H - \theta_o) \quad (4)$$

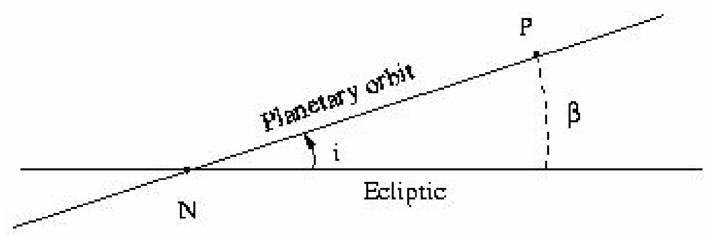

Figure 3: Heliocentric latitude of a Planet

This is also essentially the rule for calculating the latitude, as given in Indian texts, at least from the time of Āryabhaṭa. For the exterior planets, it was stipulated that

$$\theta_H = \theta_{mS} \quad (5)$$

the *mandasphuṭa-graha*, which as we saw earlier, coincides with the heliocentric longitude of the exterior planet. The same rule applied for interior planets would not have worked, because according to the traditional Indian planetary model, the *manda*-corrected mean longitude for the interior planet has nothing to do with its true heliocentric longitude. However, all the older Indian texts on astronomy stipulated that, in the case of the interior planets, the latitude is to be calculated from equation (4) with

$$\theta_H = \theta_S + \textit{manda} \text{ correction}, \quad (6)$$

the *manda*-corrected longitude of the *śīghrocca*. Since the longitude of the *śīghrocca* for an interior planet, as we explained above, is equal to the mean heliocentric longitude of the planet, equation (6) leads to the correct identification so that, even for an interior planet, θ_H in equation (4) becomes identical with the true heliocentric longitude.

Thus, we see that the earlier Indian astronomical texts did provide a fairly accurate theory for the planetary latitudes. But they had to live with two entirely different rules for calculating latitudes, one for the exterior planets (equation (5)), where the *mandasphuṭa-graha* appears and an entirely different one for the interior planets (equation (6)), which involves the *śīghrocca* of the plant, with the *manda* correction included.

This peculiarity of the rule for calculating the latitude of an interior planet was repeatedly noticed by various Indian astronomers, at least from the time of Bhāskarācārya I (c.629), who in his *Āryabhaṭīyabhāṣya* drew attention to the fact that the procedure given in *Āryabhaṭīya*, for calculating the latitude of an interior planet, is indeed very different from that adopted for the exterior planets.[6] The celebrated astronomer Bhāskarācārya II (c.1150) also draws attention to this peculiar procedure adopted for the interior planets, in his *Vāsanābhāṣya* on his own *Siddhāntaśiromaṇi*, and quotes the statement of Caturveda Pṛthūdakasvāmin (c.860 that this peculiar procedure for the interior planets can be justified only on the ground that this is what has been found to lead to predictions that are in conformity with observations.[7]

III. PLANETARY MODEL OF NĪLAKAṆṬHA SOMASUTVAN

Nīlakaṇṭha Somasutvan (c.1444-1550), the renowned Kerala astronomer, appears to have been led to his important reformulation of the conventional planetary model, mainly by the fact that it seemingly employed two entirely different rules for the calculation of planetary latitudes. As he explains in his *Āryabhaṭīyabhāṣya* [8], the latitude arises from the deflection of the planet (from the ecliptic) and not from that of a *śīghrocca*, which is different from the planet. Therefore, he argues that what was thought of as being the *śīghrocca* of an interior planet should be identified with the mean planet itself and the *manda* correction is to be applied to this mean planet, and not to the mean Sun. This, Nīlakaṇṭha argues, would render the rule for calculation of latitudes to be the same for all planets, exterior or interior.

Nīlakaṇṭha has presented his improved planetary model for the interior planets in his treatise *Tantrasaṅgraha* which, according to Nīlakaṇṭha's pupil Śaṅkara Vāriyar, was composed in 1500 AD.[9] We shall describe here, the main features of Nīlakaṇṭha's model in so far as they differ from the earlier Indian planetary model for the interior planets.[10]

In the first chapter of *Tantrasaṅgraha*, while presenting the mean sidereal periods of planets. Nīlakaṇṭha gives the usual values of 87.966 days and 224.702 days (which are traditionally ascribed to the *śīghroccas* of Mercury and Venus), but asserts that these are `*svaparyayas*', i.e. the mean revolution periods of the planets themselves.[11] As these are

[6] *Āryabhaṭīya*, with the Commentary of Bhāskara I and Someśvara, K.S. Shukla (ed.), New Delhi 1976, p.32, 247

[7] *Siddhāntaśiromaṇi* of Bhāskarācārya, with *Vāsanābhāṣya* and *Vāsanāvārttika* of Nṛsiṁha Daivajña, Muralidhara Chaturveda (ed.), Varanasi 1981, p. 402

[8] *Āryabhaṭīyam* with the *bhāṣya* of Nīlakaṇṭha Somasutvan: *Golapāda*, S.K. Pillai (ed.), Trivandrum 1957, p.8.

[9] *Tantrasaṅgraha* of Nīlakaṇṭha Somasutvan with the commentary *Laghuvivṛtti* of Śaṅkara Vāriyar, S.K. Pillai (ed.), Trivandrum 1958, p.2.

[10] For more details concerning Nīlakaṇṭha's model see, M. S. Sriram *et al*, *500 Years of Tantrasaṅgraha,* cited earlier, p.59-81.

[11] *Tantrasaṅgraha*, cited above, p.8. It is surprising that, though *Tantrasaṅgraha* was published nearly fifty years ago, this crucial departure from the conventional planetary model introduced by Nīlakaṇṭha seems to have been totally overlooked in most of the studies on Kerala Astronomy. For instance, Pingree in his

the mean heliocentric periods of these planets, the *madhyama-graha* or the mean longitude as calculated in Nīlakaṇṭha's model would be equal to the mean heliocentric longitude of the planet, for both the interior and exterior planets.

In the second chapter of *Tantrasaṅgraha*, Nīlakaṇṭha discusses the *manda* correction or the equation of centre and states[12] that this should be applied to the *madhyama-graha* as described above to obtain the *mandasphuṭa-graha*. Thus, in Nīlakaṇṭha's model, the *mandasphuṭa-graha* will be equal to the true heliocentric longitude for both the interior and exterior planets.

Subsequently, the *sphuṭa-graha* or the geocentric longitude is to be obtained by applying the *śīghra* correction. While Nīlakaṇṭha's formulation of the *śīghra* correction is the same as in the earlier planetary theory for the exterior planets, his formulation of the *śīghra* correction for the interior planets is different. According to Nīlakaṇṭha, the mean Sun should be taken as the *śīghrocca* for interior planets also, just as in the case of exterior planets. In Figure 4, P is the *manda*-corrected planet. E is the Earth and S the *śīghrocca* or the mean Sun. We have,

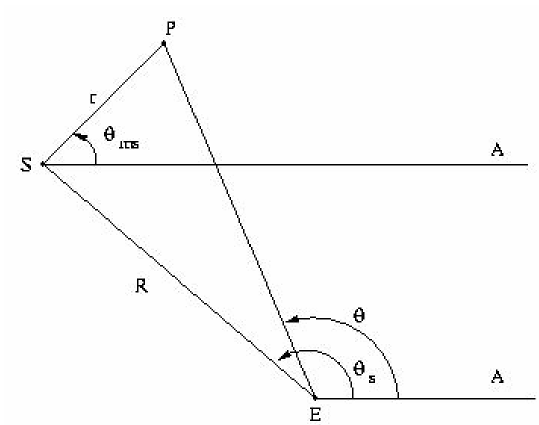

Figure 4: *Śīghra* correction for Interior Planets according to Nīlakaṇṭha

$\angle$AES = θ_S = *Śīghrocca* (mean Sun)

$\angle$ASP = θ_{mS} = *Mandasphuṭa*

$\angle$AEP = θ = True geocentric longitude of the Planet

The *śīghra-kendra* is defined in the usual way (1) as the difference between the *śīghrocca* and the *mandasphuṭa-graha*. Then from triangle ESP, we get the relation:

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review article on Indian Astronomy presents the mean rates of motion of Mercury and Venus given in *Tantrasaṅgraha* as the rates of motion of their *śīghroccas* (D.Pingree, 'History of Mathematical Astronomy in India', in *Dictionary of Scientific Biography*, Vol.XV, New York 1978, p.622).

[12] *Tantrasaṅgraha*, cited above, p.44-46.

$$\sin(\theta - \theta_S) = \frac{r \sin\sigma}{[(R + r\cos\sigma)^2 + r^2 \sin^2\sigma]^{1/2}} \tag{7}$$

which is the *śīghra* correction given by Nīlakaṇṭha for calculating the geocentric longitude of the planet. Comparing equation (7) with equations (3), and Figure 4 with Figure 2, we notice that they are the same except for the interchange of the *śīghrocca* and the *mandasphuṭa-graha*. The *manda* correction or the equation of centre is now associated with P whereas it was associated with S earlier.

In the seventh chapter of *Tantrasaṅgraha*, Nīlakaṇṭha gives formula (4) for calculating the latitudes of planets,[13] and prescribes that for all planets, both exterior and interior, θ_H in equation (4) should be the *mandasphuṭa-graha*. This is as it should be for, in Nīlakaṇṭha's model, the *mandasphuṭa-graha* (the *manda*-corrected man longitude) coincides with the true heliocentric longitude, for both the exterior and interior planets. Thus Nīlakaṇṭha, by his modification of traditional Indian planetary theory, solved the long-standing problem in Indian astronomy, of there being two different rules for calculating the planetary latitudes.

In this way Nīlakaṇṭha, by 1500 AD, had arrived at a consistent formulation of the equation of centre and a reasonable planetary model that is applicable also to the interior planets, perhaps for the first time in the history of astronomy. Just as was the case with the earlier Indian planetary model, the ancient Greek planetary model of Ptolemy and the planetary models developed in the Islamic tradition during the 8th-15th centuries postulated that the equation of centre for an interior planet should be applied to the mean Sun, rather than to the mean heliocentric longitude of the planet as we understand today. In fact, Ptolemy seems to have compounded the confusion by clubbing together Venus along with the exterior planets and singling out Mercury as following a slightly deviant geometrical model of motion.[14] Further, while the ancient Indian astronomers successfully used the notion of the *śīghrocca* to arrive at a satisfactory theory of the latitudes of the interior planets, the Ptolemaic model is totally off the mark when it comes to the question of latitudes of these planets.[15]

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[13] *Tantrasaṅgraha*, cited above, p.139.

[14] See for example, *The Almagest* by Ptolemy, Translated by G. J. Toomer, London 1984. For the exterior planets, the ancient Indian planetary model and the model described by Ptolemy are very similar except that, while the Indian astronomers use a variable radius epicycle, Ptolemy introduces the notion of an *equant*. Ptolemy adopts the same model for Venus also, and presents a slightly different model for Mercury. In both cases the equation of centre is applied to the mean Sun.

[15] As a well known historian of astronomy has remarked: “In no other part of planetary theory did the fundamental error of the Ptolemaic system cause so much difficulty as in accounting for the latitudes, and these remained the chief stumbling block up to the time of Kepler.” (J.L.E. Dreyer, *A History of Astronomy from Thales to Kepler*, New York 1953, p.200)

Even the celebrated Copernican revolution brought about no improvement in the planetary theory for the interior planets. As is widely known now, the Copernican model was only a reformulation of the Ptolemaic model (with some modifications borrowed from the Maragha School of Astronomy of Nasir ad-Din at-Tusi (c.1201-74), Ibn ash-Shatir (c.1304-75) and others) for a heliocentric frame of reference, without altering his computational scheme in any substantial way for the interior planets. As a recent study notes:

> 'Copernicus, ignorant of his own riches, took it upon himself for the most part to represent Ptolemy, not nature, to which he had nevertheless come the closest of all.' In this famous and just assessment of Copernicus, Kepler was referring to the latitude theory of Book V [of *De Revolutionibus*], specifically to the 'librations' of the inclinations of the planes of the eccentrics, not in accordance with the motion of the planet, but… by the unrelated motion of the earth. This improbable connection between the inclinations of the orbital planes and the motion of the earth was the result of Copernicus's attempt to duplicate the apparent latitudes of Ptolemy's models in which the inclinations of the epicycle planes were variable. In a way this is nothing new since Copernicus was also forced to make the equation of centre of the interior planets depend upon the motion of the earth rather than the planet.[16]

Indeed, it appears that the correct rule for applying the equation of centre for an interior planet to the mean heliocentric planet (as opposed to the mean Sun), and a satisfactory theory of latitudes for the interior planets, were first formulated in the Greco-European astronomical tradition only in the early 17th century by Kepler.

V. GEOMETRICAL MODEL OF PLANETARY MOTION

It is well known that the Indian astronomers were mainly interested in successful computation of the longitudes and latitudes of the Sun. Moon and the planets, and were not much worried about proposing models of the universe. The Indian astronomical texts, as a rule, present detailed computational schemes for calculating the Geocentric positions of the Sun, Moon and the planets. Their exposition of planetary models is by and large analytical and the geometrical picture of planetary motion does not seem to play any crucial role in their basic formulations.[17]

Detailed observations and the following sophistication of their computations of course suggested some geometrical models, and once in a while the Indian astronomers did discuss the geometrical model implied by their computations. The renowned Kerala

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[16] N.M Swerdlow and O. Neugebauer, *Mathematical Astronomy in Copernicus' De Revolutionibus*, Part I, New York 1984, p.483.

[17] The reader is referred to the discussion in the Appendix regarding the fundamental differences between the Indian and the Greco-European approaches to planetary theory.

astronomer Parameśvara of Vatasseri (c.1380-1460) has discussed the geometrical model implied in the conventional planetary model of Indian astronomy. Dāmodara the son and disciple of Parameśvara was the teacher of Nīlakaṇṭha. Nīlakaṇṭha often refers to Parameśvara as *Paramaguru*. In his super-commentary *Siddhāntadīpikā* (on Govindasvāmin's commentary on) *Mahābhāskarīya* of Bhāskarācārya-I, Parameśvara gives a detailed exposition of the geometrical picture of planetary motion as implied by the conventional model of planetary motion in Indian astronomy.[18] A shorter version of this discussion is available in his commentary *Bhaṭadīpikā* on *Āryabhaṭīya*. [19]

Following Parameśvara, Nīlakaṇṭha has also discussed in detail the geometrical model of motion as implied by his revised planetary model. Nīlakaṇṭha is very much aware that the geometrical picture of planetary motion crucially depends on the computational scheme employed for calculating the planetary positions. In his *Āryabhaṭīyabhāṣya*, Nīlakaṇṭha clearly explains that the orbits of the planets, and the various auxiliary figures such as the concentric and eccentric circles associated with the *manda* and *śīghra* processes, are to be inferred from the computational scheme for calculating the *sphuṭa-graha* (true geocentric longitude) and *vikṣepa* (latitude of the planets).[20]

Nīlakaṇṭha's revision of the traditional computational scheme for the longitudes and latitudes of the interior planets, Mercury and Venus, was based on his clear understanding of the latitudinal motion of these planets. It is this understanding which also leads him to a correct geometrical picture of the motion of the interior planets. The best exposition of this revolutionary discovery by Nīlakaṇṭha is to be found in his *Āryabhaṭīyabhāṣya*, which is reproduced below:

> Now he [Āryabhaṭa] explains the nature of the orbits and their locations for Mercury and Venus…In this way, for Mercury, the increase of the latitude occurs only for 22 days and then in the next 22 days the latitude comes down to zero. Thus Mercury moves on one side of the *apamaṇḍala* (the plane of the ecliptic) for 44 days and it moves on the other side during the next 44 days. Thus one complete period of the latitudinal motion is completed in 88 days only, as that is the period of revolution of the *śīghrocca* [of Mercury].
>
> The latitudinal motion is said to be due to that of the *śīghrocca*. How is this appropriate? Isn't the latitudinal motion of a body dependent on the motion of that body only, and not because of the motion of something else? The latitudinal motion of one body cannot be obtained as being due to the motion of another body. Hence [we should conclude that] Mercury goes around its own orbit in 88 days… However this also is not

[18] *Siddhāntaīipikā* of Parameśvara on *Mahābhāskarīyabhāṣya* of Govindasvāmin, T.S. Kuppanna Sastri (ed.), Madras 1957, p.233-238.
[19] *Bhaṭaīipikā* of Parameśvara on *Āryabhaṭīya*, H. Kern (ed.), Leiden 1874, p.60-1. It is surprising that this important commentary, published over 125 years ago, has not received any scholarly attention.
[20] *Āryabhaṭīyabhāṣya* of Nīlakaṇṭha, *Kālakriyāpāda*, K. Sambasiva Sastri (ed.), Trivandrum 1931, p.70.

> appropriate because we see it going around [the Earth] in one year and not in 88 days. True, the period in which Mercury completes one full revolution around the *bhagola* (the celestial sphere) is one year only [like the Sun]…
>
> In the same way Venus also goes around its orbit in 225 days only…
>
> All this can be explained thus: The orbits of Mercury and Venus do not circumscribe the earth. The Earth is always outside their orbit. Since their orbit is always confined to one side of the [geocentric] celestial sphere, in completing one revolution they do not go around the twelve *rāśis* (the twelve signs).
>
> For them also really the mean Sun is the *śīghrocca*. It is only their own revolutions, which are stated to be the revolutions of the *śīghrocca* [in ancient texts such as the *Āryabhaṭīya*].
>
> It is only due to the revolution of the Sun [around the Earth] that they (i.e. the interior planets, Mercury and Venus) complete their movement around the twelve *rāśis* [and complete their revolution of the Earth]… Just as in the case of the exterior planets (Jupiter etc.), the *śīghrocca* (i.e., the mean Sun) attracts [and drags around] the *manda-kakṣyā-maṇḍala* (the *manda* orbits on which they move) in the same way it does for these [interior] planets also.[21]

The above passage exhibits the clinching argument employed by Nīlakaṇṭha. From the fact that the motion of the interior planets is characterized by two different periods, one for their latitudinal motion and another for their motion in longitude, Nīlakaṇṭha arrived at what may be termed a revolutionary discovery concerning the motion of the interior planets: That they go around the Sun in orbits that do not circumscribe the Earth in a period that corresponds to the period of their latitudinal motion (which is the period assigned to their *śīghrocca s* in the traditional planetary model), and that they go around the zodiac in one year as they are dragged around the Earth by the Sun.

It was indeed well known to the ancients that the exterior planets, Mars, Jupiter and Saturn, go around the Earth and they also go around the Sun in the same mean period, because their geocentric orbit is outside that of the Sun. Nīlakaṇṭha was the first savant in the history of astronomy to clearly derive from his computational scheme, and not from any speculative or cosmological argument, that the interior planets go around the Sun and the period of their motion around Sun is also the period of their latitudinal motion. The

[21] *Āryabhaṭīyabhāṣya* of Nīlakaṇṭha, *Golapāda*, cited above, p. 8-9.

fact that the mean period of their motion in longitude around the Earth is the same as that of the Sun is explained as being due to their being carried around the Earth by the Sun.

Nīlakaṇṭha also wrote a tract called *Grahasphuṭānayane vikṣepavāsanā*, where he has set forth his latitude theory in detail. There he has given the qualitative nature of the orbits of the Sun, Moon and the five Planets in a single verse, which may be cited here:

> The Moon and the Planets are deflected along their *manda-kakṣyā* (*manda* orbit) from the ecliptic both to the North and the South by amounts depending on their [longitudinal] separation from their nodes. For the Moon the centre of *manda-kakṣyā* is also the centre of the ecliptic. For Mars and other planets, the centre of their *manda-kakṣyā* [which is also the centre of their *manda* deferent circle], is the mean Sun that lies on the orbit of the Sun on the ecliptic. [22]

Nīlakaṇṭha presents a clear and succinct statement of the geometrical picture of the planetary motion as implied by his revised planetary model in two of his small tracts, *Siddhāntadarpaṇa* and *Golasāra*. We present the version given in *Siddhāntadarpaṇa*:

> The [eccentric] orbits on which planets move (*graha-bhramaṇa-vṛtta*) themselves move at the same rate as the apsides (*uccha-gati*) on *manda-vṛtta* [or the *manda* epicycle drawn with its centre coinciding with the centre of the *manda* concentric]. In the case of the Sun and the Moon, the centre of the Earth is the centre of this *manda-vṛtta*.
>
> For the others [namely the planets Mercury, Venus, Mars, Jupiter and Saturn] the centre of the *manda-vṛtta* moves at the same rate as the mean Sun (*madhyārka-gati*) on the *śīghra-vṛtta* [or the *śīghra* epicycle drawn with its centre coinciding with the centre of the *śīghra* concentric]. The *śīghra-vṛtta* for these planets is not inclined with respect to the ecliptic and has the centre of the celestial sphere as its centre.
>
> In the case of Mercury and Venus, the dimension of the *śīghra-vṛtta* is taken to be that of the concentric and the dimensions [of the epicycles] mentioned are of their own orbits. The *manda-vṛtta* [and hence the *manda* epicycle of all the planets] undergoes increase and decrease in size in the same way as the *karṇa* [or the hypotenuse or the distance of the planet from the centre of the *manda* concentric]. [23]

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[22] *Grahasphuṭānayane vikṣepavāsanā* of Nīlakaṇṭha, in *Gaṇitayuktayaḥ*, K. V. Sarma (ed.), Hoshiarpur 1979, p.63

[23] *Siddhāntadarpaṇa* of Nīlakaṇṭha, K. V. Sarma (ed.), Hoshiarpur 1976, p. 18.

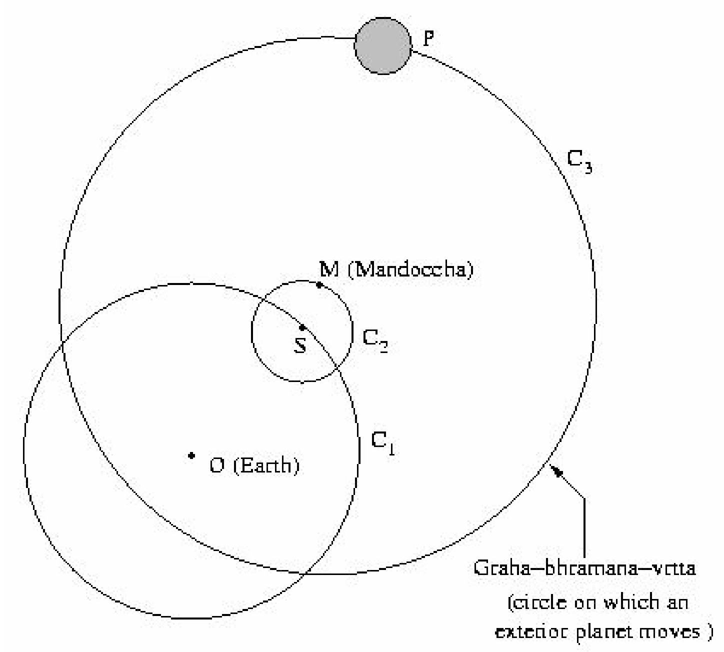

Figure 5: Nīlakaṇṭha's geometrical model for an Exterior Planet

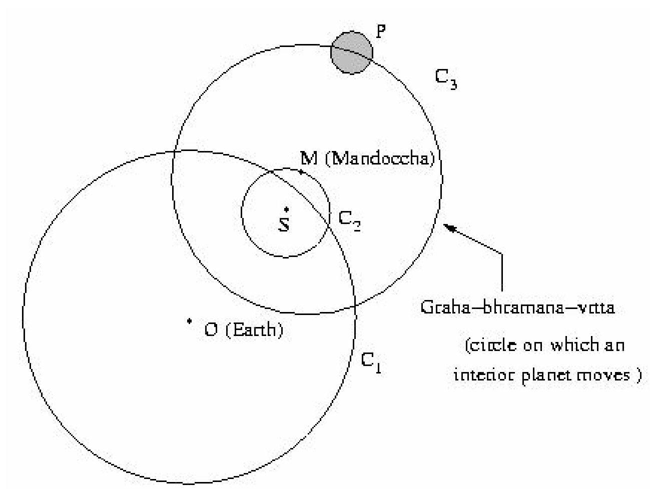

Figure 6: Nīlakaṇṭha's geometrical model for an Interior Planet

The geometrical picture described above is presented in Figures 5, 6. It is important to note that Nīlakaṇṭha has a unified model for both the exterior and interior planets and the same is reflected in his formulation of the corresponding geometrical picture of planetary motion. Nīlakaṇṭha's description of the geometrical picture of the planetary motions involves the notions of *manda-vṛtta* and *śīghra-vṛtta*, which are nothing but the *manda* and *śīghra* epicycles drawn with the centre of their concentric as the centre. These concepts are explained clearly in the beginning of the eighth chapter of the celebrated Malayalam treatise on mathematical astronomy *Yuktibhāṣā* of Jyeṣṭhadeva (c.1530) who was a junior contemporary of Nīlakaṇṭha.

An important point to be noted is that the geometrical picture of planetary motion as discussed above, deals with the orbit of each of the planets individually and does not put them together in a single geometrical model of the planetary system. Each of the exterior planets have different *śīghra-vṛttas*, which is in the same plane as the ecliptic, and we have to take the point where the *āditya-sūtra* (the line drawn from the centre along the mean Sun) touches each of these *śīghra-vṛttas* as the centre of their *manda-vṛtta.* On this *manda-vṛtta* the *mandocca* is to be located, and with that as the centre the *graha-bhramaṇa-vṛtta* or the planetary orbit is drawn with the standard radius (*trijyā* or Rsin90). In the case of the interior planets, Nīlakaṇṭha says that the *śīghra-vṛtta* has to be drawn with the standard radius (*trijyā* or Rsin90) and the *graha-bhramaṇa-vṛtta* is to be drawn with the given value of the *śīghra* epicycles as the radii. In this way, we see that the two interior planets can be represented in the same diagram, as the *śīghra-vṛtta* is the same for both of them.

To integrate the diagrams for all the planets into a single diagram of the planetary system, we shall have to use the notion of *bhū-tārāgraha-vivara* or the earth-planet distance. Nīlakaṇṭha has discussed this extensively in his *Āryabhaṭīyabhāṣya* and has shown how the effects of the latitudinal motions of the planets should be taken into account in the computation of the earth-planet distance. The final diagram that we would obtain, by putting all planets together in a single diagram adopting a single scale, is essentially what Nīlakaṇṭha has described as the qualitative picture of planetary motion that we presented earlier: The five planets, Mercury, Venus, Mars, Jupiter, and Saturn move in eccentric orbits around the mean Sun, which goes around the Earth. The planetary orbits are tilted with respect to the orbit of the Sun or the ecliptic, and hence cause the motion in latitude. Since it is well known that the basic scale of distances are fairly accurately represented in the Indian astronomical tradition, as the ratios of the radius of the *śīghra* epicycle to the radius of the concentric (*trijyā*) is very nearly the mean ratio of the Earth-Sun and the Earth-Planet distances (for exterior planets) or the inverse of it (for interior planets), the planetary picture will also be fairly accurate in terms of the scales of distances.

Nīlakaṇṭha's modification of the conventional planetary model of Indian astronomy seems to have been adopted by most of the later astronomers of the Kerala School. This is not only true of Nīlakaṇṭha's pupils and contemporaries such as Citrabhānu (c.1530), Śaṅkara Vāriyar (c.1500–1560) and Jyeṣṭhadeva (c.1500–1600)[24], but also of later astronomers such as Acyuta Piṣāraṭi (c.1550-1621), Putumana Somayāji (c.1660– 1740) and others. Incidentally, it may be of interest to note that the well-known Oriya astronomer of 19th century, Candraśekhara Sāmanta, who was trained solely in traditional Indian astronomy, wrote a treatise *Siddhāntadarpaṇa*, in 1869, wherein he has also discussed a model of planetary motion in which the five planets, Mercury, Venus, Mars, Jupiter and Saturn, go around the Sun.[25]

[24] The Malayalam work *Yuktibhāṣā* of Jyeṣṭhadeva gives a detailed exposition of the planetary model introduced in *Tantrasaṅgraha*, apart from presenting detailed rationale for all the processes outlined therein.

[25] *Siddhāntadarpaṇa*, of Candraśekhara Sāmanta, J.C.Roy (ed.), Calcutta 1897, verse V.36.

Modern scholars of Indian astronomical tradition have noted that the Indian astronomers were mainly interested in successful computation of the longitudes and latitudes of the Sun, Moon and the planets, and were not much concerned about proposing models of the universe. The Indian astronomical texts, as a rule, present detailed computational schemes for calculating the geocentric positions of the Sun and Moon and the planets. Their exposition of planetary models is by and large analytical and the geometrical picture of planetary motion does not play any crucial role in their basic formulations.

Sometimes, the Indian texts of astronomy also include a discussion of the geometrical picture of planetary motion as implied by their computational schemes. As we noted earlier, Parameśvara (c.1380-1460), the *Paramaguru* of Nīlakaṇṭha, presented a detailed exposition of the geometrical picture of planetary motion as implied by the traditional planetary model employed by the Indian astronomers, at least since the time of Āryabhaṭa (499 AD). Following this, Nīlakaṇṭha (c.1444-1550) discussed the geometrical picture of planetary motion that is implied by his own revised planetary model. According to Nīlakaṇṭha, the five Planets - Mercury, Venus, Mars, Jupiter and Saturn - move in eccentric orbits around the mean Sun, which in turn goes around the Earth.

The geometrical picture of planetary motion as outlined by Nīlakaṇṭha does seem similar to the model of planetary motion which was proposed nearly a century later by the European astronomer Tycho Brahe (c.1583). However, Nīlakaṇṭha's fairly accurate understanding of the geometrical orbit of the planets does not arise in the course of any speculative debate concerning the relative merits of heliocentric and geocentric cosmologies. Indeed, the outstanding achievements of Nīlakaṇṭha and Tycho Brahe belong to different traditions of astronomy. The motivation and the spirit behind their geometrical models of planetary motion, and the way they arrive at them, all seem to be profoundly different. To understand the work of Nīlakaṇṭha in the proper perspective it is essential to have some idea of the basic difference in approach between the Greco-European tradition in Astronomy and the Indian tradition in Astronomy, especially as regards planetary theory.

The Greek Approach to Planetary Theory as Expounded in Ptolemy's Almagest

One of the best sources to study the Greek approach to planetary theory is the great work of Claudius Ptolemy (c.150 AD), *The Mathematical Syntaxis*, more popularly known by its Arabic name, *The Almagest*, which contains the most systematic exposition of Greek mathematical astronomy. In the first section of *The Almagest*, Ptolemy summarises the Aristotelian classification of natural philosophy into physics, mathematics and theology. Of these, physics, which dealt with the "corruptible bodies…below the lunar sphere", could never be an exact discipline worthy of philosophers' attention; and theology, which dealt with "the first cause of the first motion of the universe…is completely separated from perceptible reality." Only mathematics, which concerned itself with "eternal things with an ethereal nature", the "divine and heavenly things", can provide "sure and unshakeable knowledge to its devotees". In essence, mathematics, or the study of motion of the celestial objects above the lunar sphere, alone was worthy of philosophers' attention for that alone is characterised by eternal unchanging laws. In Ptolemy's own words:

> For Aristotle divides theoretical philosophy too, very fittingly, into three primary categories, physics, mathematics and theology. For everything that exists is composed of matter, form and motion; none of these [three] can be observed in its substratum by itself, without the others: they can only be imagined. Now the first cause of the first motion of the universe, if one considers it simply, can be thought of as an invisible and motionless deity; the division [of theoretical philosophy] concerned with investigating this [can be called] 'theology', since this kind of activity, somewhere up in the highest reaches of the universe, can only be imagined, and is completely separated from perceptible reality. The division [of theoretical philosophy] which investigates material and ever-moving nature, and which concerns itself with 'white', 'hot', 'sweet', 'soft' and suchlike qualities one may call 'physics'; such an order of being is situated (for the most part) amongst corruptible bodies and below the lunar sphere. That division [of theoretical philosophy] which determines the nature involved in forms and motion from place to place, and which serves to investigate shape, number, size, and place, time and suchlike, one may define as 'mathematics'. Its subject-matter falls as it were in the middle between the other two, since, firstly, it can be conceived of both with and without the aid of the senses, and, secondly, it is an attribute of all existing things without exception, both mortal and immortal: for those things which are perpetually changing in their inseparable form, it changes with them, while for eternal things which have an ethereal nature, it keeps their unchanging form unchanged.
>
> From all this we concluded: that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them; and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we are drawn to the investigation of that part of theoretical philosophy, as far as we were able to the whole of it, but especially to the theory concerning the divine and heavenly things. For that alone is devoted to the investigation of the eternally unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. [26]

[26] *The Almagest*, cited earlier, p.36-7.

In the third section of Book I of *The Almagest*, Ptolemy goes on to explain that the celestial bodies, being constituted of the ideal substance "ether", are endowed with the ideal shape, namely that of a sphere; they undergo only ideal motion, namely uniform circular motion:

> The ether is, of all bodies, the one with constituent parts which are finest and most like each other; now bodies with parts like each other have surfaces with parts like each other; but the only surfaces with parts like each other are the circular, among the planes, and the spherical among the three-dimensional surfaces. And since the ether is not plane, but three-dimensional, it follows that it is spherical in shape. Similarly, nature formed all earthly and corruptible bodies out of shapes which are round but of unlike parts, but all ethereal and divine bodies out of shapes which are of like parts and spherical. For if they were flat or shaped like a discuss they would not always display a circular shape to all those observing them from simultaneously from different places on earth. For this reason it is plausible that the ether surrounding them, too, being of the same nature, is spherical, and because of the likeness of its parts moves in a circular and uniform motion. [27]

Ptolemy takes up the subject of planetary motion in Book IX of *The Almagest*. In the second section he enunciates the basic hypothesis that their motion, like that of the sun and the moon, ought to be "represented by uniform circular motions", as that is what is proper for these "divine beings". In Ptolemy's words:

> Now it is our purpose to demonstrate for the five planets, just as we did for the sun and moon, that all their apparent anomalies can be represented by uniform circular motions, since these are proper to the nature of divine beings, while disorder and non-uniformity are alien [to such beings]. Then it is right that we should think success in such a purpose a great thing, and truly the proper end of mathematical part of theoretical philosophy. But, on many grounds, we must think that it is difficult, and there is good reason why no one before us has yet succeeded in it…
>
> Hence it was, I think, that Hipparchus, being a great lover of truth, for all the above reasons, and especially because he did not yet have in his possession such a ground-work of resources in the form of accurate observations from earlier times as he himself has provided to us, although he investigated the theories of the sun and moon, and, to the best of his ability, demonstrated with every means at his command that they are represented by uniform circular motions, did not even make a beginning in establishing theories for the five planets, not at least in his writings which have come down to us. All that he did was to make a compilation of the

[27] *The Almagest,* cited earlier, p. 40.

> planetary observations arranged in a most useful way, and to show by means of these that the phenomena were not in agreement with the hypotheses of the astronomers of that time. [28]

To some extent the above extracts from Almagest summarise the basic approach to astronomy that prevailed in the Greco-European tradition till about the end of sixteenth century.

The Indian Approach to Planetary Theory

The Indian texts of Astronomy, or *Jyotiḥśāstra*, present as the main *prayojana* or the *raison de etre* of the *śāstra* to be the determination of *kāla* (time), *dik* (direction) and *deśa* (place). The ancient *Vedāṅga-jyotiṣa* texts declare *Jyotiḥśāstra* to be *kāla-vidhāna-śāstra*, the science of determining time. One of the standard texts of *Jyotiḥśāstra*, *Siddhāntaśiromaṇi* of Bhāskarācārya (c.1150 AD) states that "from this [*Jyotiḥ*] *śāstra* there arises *kāla-bodha*, the knowledge of time". And, his commentator Nṛsiṁha Daivajña (c.16th Century) explains, "that the term *kāla* also encompasses *dik*". Now, the determination of *kāla*, *dik* and *deśa* is to be achieved through *grahagati-parīkṣā*, a study of the motion of the celestial objects.[29]

Thus, the object of *Jyotiḥśāstra* was not to discover the true cosmological model of the universe, or even the true laws of planetary motion; it was the more mundane one of determining time, direction and space accurately by a careful study of the motion of the celestial bodies. For this purpose, the Indian astronomers put all their efforts in making accurate observations, developing suitable theories and efficient methods of calculation, and evolving critical tests to help them correct their theories whenever their calculations failed to correspond with observation.

Further, the Indian Astronomical texts repeatedly emphasise that *śāstras* become *slatha*, inaccurate, over time. This is taken to be inherent in the very nature of things, although, sometimes, detailed reasons are given as to why many great *śāstras* of ancient times have become inadequate. The indication that a *śāstra* has become *slatha* is almost always found in the failure to achieve *dṛg-gaṇitaikya*, concordance between calculation and observation. And whenever a *śāstra* becomes *slatha*, the Astronomers are expected to undertake *śāstra-saṁsthāpana*, careful re-examination of their theories leading to revision of the various procedures and parameters used in them. Many a time this would have proved to be too daunting a task. Commenting on the faint-heartedness of some of his predecessors, Nīlakaṇṭha declares in his seminal work on the philosophical foundations of the science of Astronomy, *Jyotirmīmāṁsā*:

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[28] *The Almagest*, cited earlier, p. 420-1.
[29] *Siddhāntaśiromaṇi* of Bhāskarācārya, with *Vāsanābhāṣya* and *Vāsanāvārttika* of Nṛsiṁha Daivajña, cited above, p.10-11.

> A commentator on the *Mānasa* [*Laghumānasa* of *Muñj*āl*a*] has lamented: 'Indeed, the *siddhāntas*, like *Paitāmaha*, differ from one another [in giving the astronomical constants]. Timings are different as the *siddhāntas* differ [i.e. the measures of time at a particular moment differ as computed by the different *siddhāntas*]. When the computed timings differ, Vedic and domestic rituals, which have [correct] timings as a component [of their performance] go astray. When rituals go astray, worldly life gets disrupted. Alas, we have precipitated into a calamity.'
>
> Here, it needs to be stated: 'O faint-hearted, there is nothing to be despaired of. Wherefore does anything remain beyond the ken of those intent on serving at the feet of the teachers [and thus gain knowledge]? One has to realise that the five *siddhāntas* had been correct at a particular time. Therefore, one should search for a *siddhānta* that does not show discord with actual observations [at the present time]. Such accordance with observation has to be ascertained by observers during the times of eclipses etc. When *siddhāntas* show discord, i.e., when an early *siddhānta* is in discord, observations should be made of revolutions etc. [which would give results, which accord with actual observation] and a new *siddhānta* enunciated.' [30]

Earlier, in the same work, Nīlakaṇṭha has an interesting comment on the view that all *śāstras* are divine revelations and hence are not subject to any corrections or revisions. He states:

> Some say that Brahmā who was pleased with the penance (*tapas*) of Āryabhaṭa gave him instruction regarding the planetary revolutions, [epicycle] circumferences etc., essential for calculating the motion of planets. Āryabhaṭa has put down these instructions faithfully in his *daśagītikā* [part of *Āryabhaṭīya* which gives the parameters of planetary theory]. How can this be subjected to further test and revision, as Brahma indeed is omniscient (*sarvajña*), free of all passions... Oh dumb-witted! This is not so. The divine grace (*devatā-prasāda*) is only for attaining clarity of intellect. Nor is it the case that Brahmā or the Sun God would Himself come and give instruction. [31]

It is this understanding of the *śāstra* as an essentially human construct (*puruṣa-buddhi-prabhava*) that enables Indian scientists to reconcile and live with several schools of thought (*siddhāntas* or *pakṣas*) in any *śāstra* as long as they are found adequate in practice. If the purpose of *Jyotiḥśāstra* were to arrive at the true picture of the heavens, then when Āryabhaṭa proposed the model of diurnal rotation of earth as opposed to the (then) traditional model of the rotation of the celestial sphere, all work in *Jyotiḥśāstra* would have focused only on resolving which of the two models was indeed the 'true' one.

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[30] *Jyotirmīmāṁsā* of Nīlakaṇṭha, K.V.Sarma (ed.), Hoshiarpur 1977, p.6.
[31] *Jyotirmīmāṁsā,* cited above, p. 2.

Instead, Indian astronomers of both schools continued to concentrate on refining basic astronomical parameters and computational schemes in order to arrive at better accord with observations. Settling what constitutes a true picture of the world was surely not the *raison-de-etre* of their science.

As regards the epistemological status of the planetary models, the Indian astronomical texts present a very clear position that they are conceptual tools, which serve the purpose of calculating observationally verified planetary positions. Notions such as the apsides (*ucca, nīca*), mean (*madhyama*), eccentrics or epicycles used in *manda* and *śīghra* corrections (*manda-paridhi* etc.) and so on – notions which are employed in various planetary models – are all conceptual constructs and there are no constraints on our choice of them except that the model should lead to results in concordance with observations. This principle is clearly set out for instance in the famous *Āryabhaṭīya-bhāṣya* of Bhāskarācārya I (c. 629 AD), when he starts his exposition of planetary models based on the *manda* and *śīghra* corrections:

> There are no constraints or limitations imposed on the aids such as the *ucca*, *nīca*, *madhyama*, *paridhi* and so on which are indeed aids to the calculation of the observed motion of planets. These are indeed but means for arriving at the desired results. Hence this entire procedure is fictitious, by means of which the observed planetary motion is arrived at. Just as the seekers of ultimate knowledge expound the ultimate truth via untrue means; just as the surgeons practice their surgery etc. on stems and other objects; just as the hair-stylists practice shaving on pots; just as the experts in performance of *yajña* practice using dry wood; just as the linguists utilise notions such as *prakṛti*, *pratyaya*, *vikāra*, *āgama*, *varṇa*, *lopa*, *vyatyaya*, etc., to derive (well formed) words; in the same way in our science also the astronomers employ notions such as *madhyama*, *mandocca*, *śīghrocca*, *śīghra-paridhi*, *jyā, kaṣṭha*, *bhujā*, *koṭi*, *karṇa*, etc., in order to derive the observed motion of planets. Hence, there is indeed nothing unusual that fictitious means are employed to arrive at the true state of affairs [in all these sciences]. [32]

There is a very similar statement made by the renowned Astronomer Caturveda Pṛthūdakasvāmin (c.865AD) in his celebrated commentary on *Brahmasphuṭasiddhānta* of Brahmagupta:

> Just as the grammarians employ fictitious entities such as *prakṛti, pratyaya, āgama, lopa, vikāra*, etc. to decide on the established real word forms, and just as the *vaidyas* employ tubers etc. to demonstrate surgery, one has to understand and feel contented that it is in the same way that the astronomers postulate measures of the earth etc. and models of motion of

[32] *Āryabhaṭīyabhāṣya* of Bhāskarācārya I, cited above, p.217.

> the planets in *manda* and *śīghra-pratimaṇḍalas* for the sake of accurate predictions. [33]

In his *Āryabhaṭīyabhāṣya*, Nīlakaṇṭha also repeats the same epistemological principle that there are indeed no constraints or requirements that need to be imposed on theoretical models or procedures except that they have to lead to valid results. He goes on to quote the famous verse of the grammarian philosopher Bhartṛhari, which propounds this view:[34]

> *upādāyāpi ye heyāstānupāyān pracakṣate*
> *upāyānāñca niyamo nāvaśyamavatiṣṭhate*

The above discussion should make it amply clear that the Indian astronomers adopted an extraordinarily flexible and pragmatic view on the nature and purpose of planetary models. They were not constrained by any metaphysical presuppositions regarding the celestial bodies or the ideal motions that they ought to follow. Since the Indian astronomical tradition was also informed with the understanding that the motions of the heavenly bodies are fairly complex, it refrained from making any tall claims about the ultimate laws governing the heavens, but at the same time allowed for a high degree of flexibility and sophistication in the computational schemes that were to be employed for describing the planetary motions. These computational schemes were presented in an analytical manner, but many steps involved had fairly simple geometrical interpretation. Such geometrical interpretations were frequently presented in the Indian astronomical texts, but there was often the cautionary note that the reality was far more complex than implied by such simple geometrical pictures. This is in marked contrast with the kind of approach that characterised the development of the Greco-European tradition of astronomy till indeed the modern times.

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[33] *Vāsanābhāṣya* of Pṛthudakasvāmin on *Brahmasphuṭasiddhānta* of Brahmagupta, cited by Nṛsiṁha Daivajña in his *Vāsanāvārttika* on Bhāskarācārya's *Siddhāntaśiromaṇi*, cited above, p.48.

[34] *Āryabhaṭīyabhāṣya* of Nīlakaṇṭha, *kālakriyāpāda*, cited above, p.41. Here Nīlakaṇṭha is citing Bhartṛhari's *Vākyapadīya*, *Padakhaṇḍa*, verse 38.